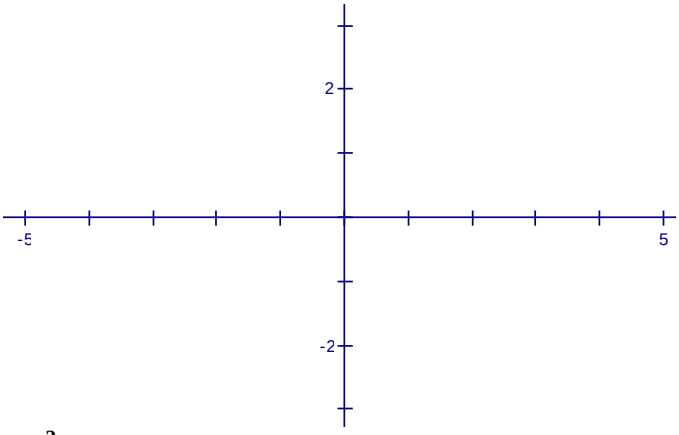
Quadratic Functions and Their Graphs

Quadratic Function (pg. 540 – 541)

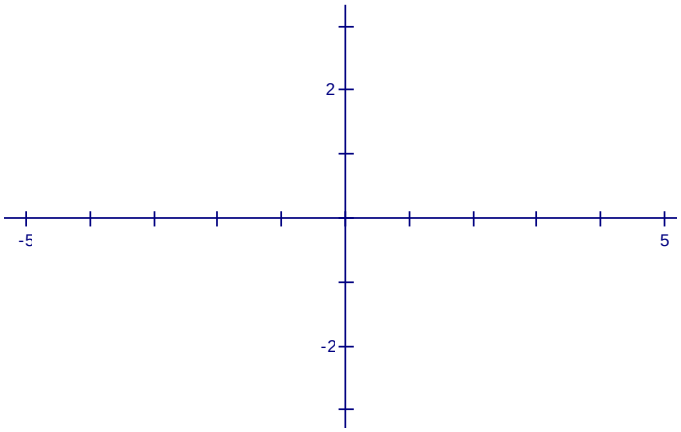
A quadratic function is a function that can be written in the form f(x) = $ax^2 + bx + c$, where *a*, *b*, and *c* are real numbers and $a \neq 0$.

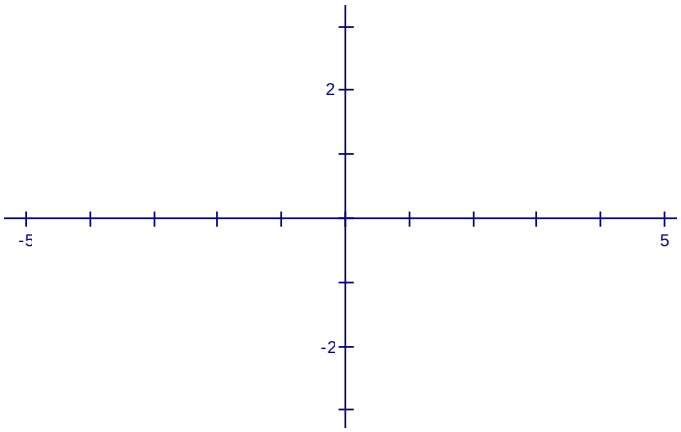
Graph the Parabola defined by $f(x) = x^2$



Graphing the Parabola Defined by $f(x) = ax^2$

If *a* is positive, the parabola opens upward, and if *a* is negative the parabola opens downward
If $|a| > 1$ the graph of the parabola is narrower than the graph of $y = x^2$
If $|a| < 1$ the graph of the parabola is wider than the graph of $y = x^2$





Graph: $f(x) = 2x^2$
Graph: $f(x) = -1x^2$
Graph: $f(x) = .5x^2$

Graphing the Parabola Defined by $f(x) = (x - h)^2$ (pg. 542 -543)

If *h* is positive, the graph of $f(x) = (x - h)^2$ is the graph of $y = x^2$ shifted to the right *h* units.

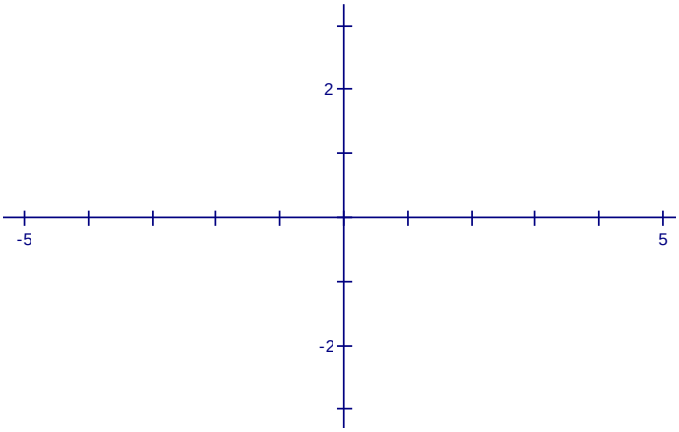
If *h* is negative, the graph of $f(x) = (x - h)^2$ is the graph of $y = x^2$ shifted to the left $|h|$ units.

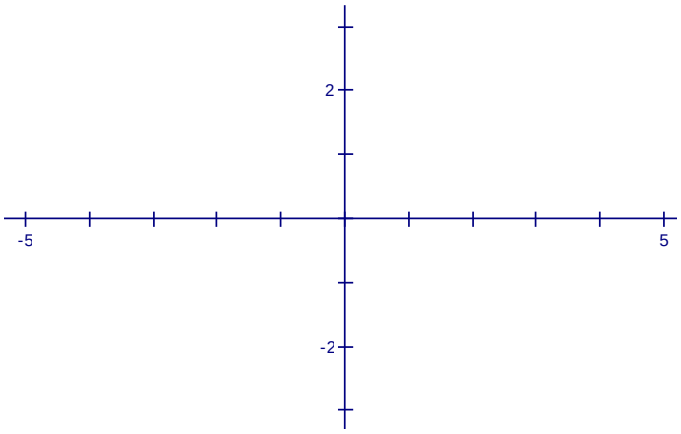
The vertex is $(h, 0)$ and the axis of symmetry is the vertical line $x = h$

Graph: $f(x) = (x - 3)^2$

Graph: $f(x) = (x + 2)^2$

Graph: $f(x) = (x - 3)^2$





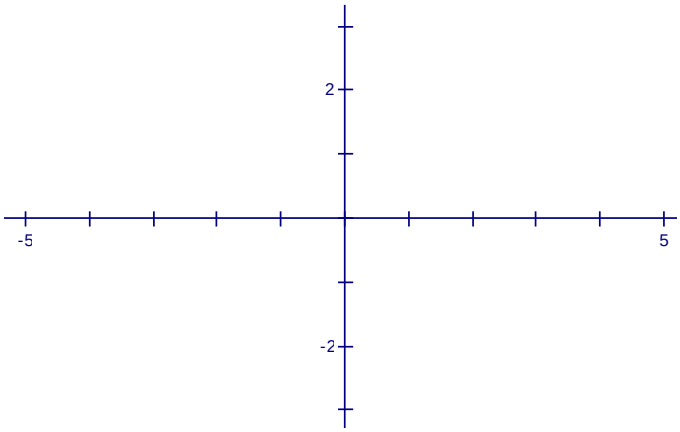
Graph the Parabola defined by $f(x) = x^2 + k$

If k is positive, the graph of $f(x) = x^2 + k$ is the graph of $y = x^2$ shifted upward k units.

If k is negative, the graph of $f(x) = x^2 + k$ is the graph of $y = x^2$ shifted downward $|k|$ units.

The vertex is $(0, k)$ and the axis of symmetry is the y-axis.

Graph: $f(x) = (x - 3)^2 - 5$

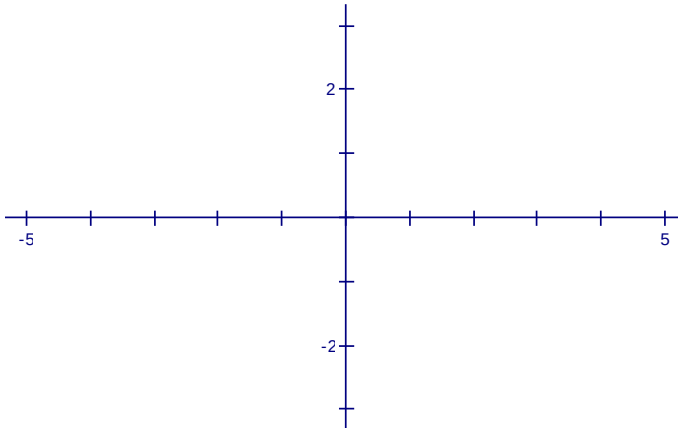


Graph the Parabola Defined by $f(x) = a(x - h)^2 + k$ (pg. 544- 545)

The parabola has the same shape as $y = x^2$

The vertex is (h, k) and the axis of symmetry is the vertical line $x = h$.

Graph: $y = -2(x + 3)^2 + 4$

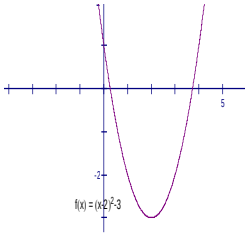


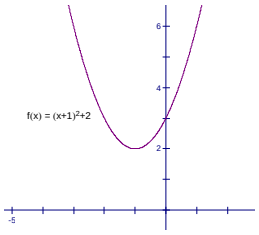
8.7 notes math 1010

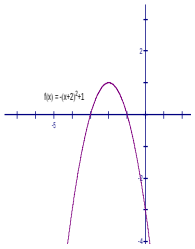
More about Graphing of Quadratic Functions

Objectives: Complete the square and find intercepts

1. Write quadratic functions in the form $\boldsymbol{y = (x - h)^2 + k}$ by completing the square. This puts the equation in vertex form. The axis of symmetry is $x = h$ (the x-coordinate of the vertex).
2. Learn the formula for finding the vertex of a parabola without completing the square.
3. Find the minimum or maximum of a quadratic function. The y-coordinate of the vertex is the minimum if the parabola opens up and the maximum if the parabola opens down.





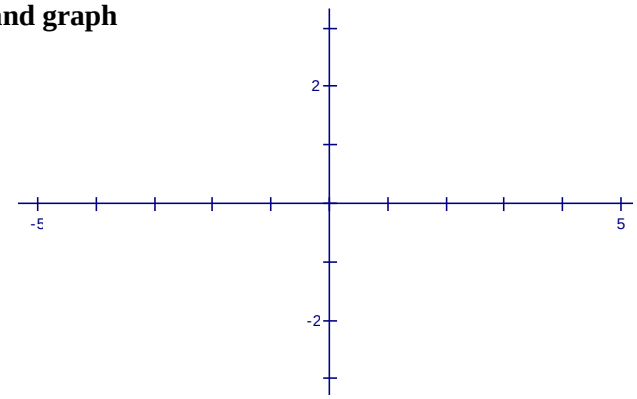


Steps for completing the square are:

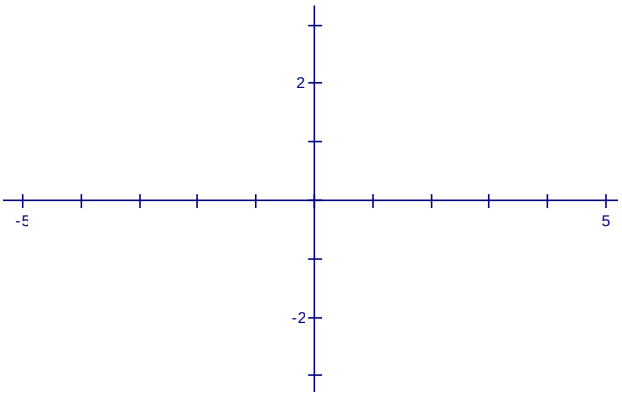
Step 1: Write the function in descending order.
Step 2: Add the constant to the other side of the equation.
Step 3: Take half the x-coefficient and square it.
Step 4: Add the number obtained in Step 3to both sides of the equation.
Step 5: Factor the side with the variables into the perfect square trinomial.
Step 6: Add the constant back to the side with the perfect square trinomial.

Examples (pg. 549 – 550): Complete the square and graph

$g(x) = x^2 - 6x + 4$

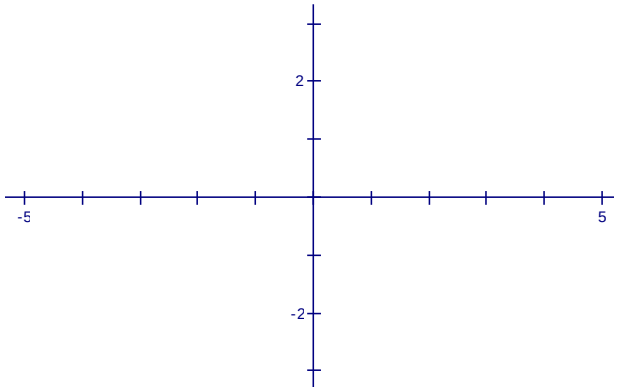


Complete the square and graph: $f(x) = x^2 + 5x + 3$

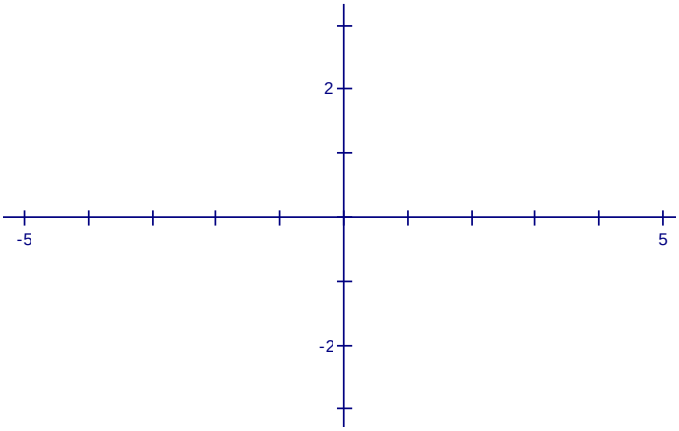


If the leading coefficient is not 1, factor out that number from the first two terms. Then complete the square and use the distributive law.

Example: Graph: $f(x)=3x^2+12x+13$ Label the vertex and the axis of symmetry.



Graph $f(x)=-2x^2+10x-7$ Find the maximum or minimum function value.



Vertex Formula (pg. 551 – 552)

The graph of f(x) = ax^2+bx+c , when a≠ 0, is a parabola with vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ or $\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$.

The x-coordinate of the vertex is $\frac{-b}{2a}$

The axis of symmetry is $x=\frac{-b}{2a}$

The second coordinate of the vertex is most commonly found by computing $f\left(\frac{-b}{2a}\right)$

To find the *y*-intercept, let *x* = 0

To find the *x*-intercepts, let *y* or *f*(*x*) = 0. You can use the quadratic formula, square root property, completing the square or factoring to find the *x*-intercept. The *x*-intercept is also called a solution, a root or a zero of the function.

Example: Find any *x*-intercepts and the *y*-intercept of the graph of $f(x) = x^2 + -2x - 2$